Trends in
Applied Sciences
Research

TASR

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Stochastic Model of Market Assessments of Stock Returns and Value of Asset Prices in Time-Varying Investment Returns

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ABSTRACT
In stock trading, investors are mainly interested in the basic decisions that will help to assess good stock return rates to optimize the value of asset pricing. Thus, in this study, a coupled system of variable coefficient problems with stochastic parameters in the model for two different investments in the capital market has been studied. The problems were solved by adopting the Frobenius method of series solution and the method of the undetermined coefficient to determine the assessments of stock returns and value of asset prices in time-varying investment returns. These equations were solved as a coupled investment equation to establish the behavior of stock return rates and asset price valuation when it follows multiplicative, additive effects series with quadratic functions which is not found in the literature. This paper extends the works in literature by incorporating multiplicative effect series and additive effects series and solving the problem as coupled systems.

KEYWORDS
Asset returns, stock market, differential equation, multiplicative effect series, additive effect series

INTRODUCTION

In capital investments long and short terms, investment plans are very crucial to investors because they plan ahead of time so that their levels of return rates will always be on the increase. Having such plans certain decisions need to be put in place in other to adequately optimize good profit margins from assets. An asset is one of the key factors an investor would not want to play with due to its benefits in the life of every trader. Therefore, the valuation of an asset is an act to assess the market value of asset prices. It has become a crucial factor in driving economic fluctuations, allocating economic resources across sectors with time and also influencing the financial strength of the entire system which yields levels of returns.

Nevertheless, considering the above issue needs a differential equation that is dynamic in harnessing different components into a simple system and to handle such analysis also needs an analytic method, which can give exact solutions for proper mathematical estimates. It is essential to study asset valuation-related problems, well formulated and accurate analytical solutions to assess the realistic valuation of stock returns, hence, the analytical solution is implemented based on the definite characteristics of the problem under study.

ISSN: 2151-7908 (Online)
ISSN: 1819-3579 (Print)
https://doi.org/10.3923/tasr.2023.50.62

Received: 24 Jul. 2022
Accepted: 01 Feb. 2023
Published: 30 Jun. 2023

In the forgoing, so many scholars have written extensively to address stock price changes. For instance, In the problem of variation of stock price changes, an analytical solution was obtained that determined equilibrium prices[1]. The Elzaki transform was used to solve ordinary differential equations with variable coefficients[2], in another manner, the Elzaki transform was applied in solving linear volterra integral equations of the first kind[3]. Superlative analysis of Mhand, Aboodh and Elzaki transforms had also been considered and the result showed little distinction from some certain partial differential equation[4]. In applying a differential equation in a cylindrical channel problem, the problem was formulated and solved analytically by the Frobenius method and the solution was obtained in Bessel functions[5].

However, lots of scholars have written extensively on stock market prices[6-9]. While stability analysis of a stochastic model of price changes at the floor of a stock market has also been studied[10]. In their research précised conditions are obtained which determine the equilibrium price and growth rate of stock shares. The stochastic analysis of the behavior of stock prices was examined[11]. Results showed that the proposed model is efficient for the prediction of stock prices. Another work looked at the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE)[12], in their research, the drift and volatility coefficients for the stochastic differential equations were determined. The geometric Brownian Motion and study of the accuracy of the model with detailed analysis of simulated data had also been carried out[13].On the other hand, a stochastic model of price changes at the floor of the stock market was considered[14]. In their research, the equilibrium price and the market growth rate of shares were determined.

In this study, a coupled system of 2nd ordinary differential equations with stochastic parameters was studied. These equations were solved as a coupled investment equation to establish the behavior of stock return rates and asset price valuation when it follows multiplicative, additive effects series with quadratic functions which was not considered by previous efforts. This paper extends the works in literature Osu[1], by incorporating multiplicative effect series and additive effects series and solving the problem as coupled systems[13].

This study aimed to propose an investment equation whose rate of return follows multiplicative and additive effects series respectively in time-varying investment returns. To the best of our knowledge, this is the first study that has combined fully coupled investment equations with a detailed analytical solution.

MATHEMATICAL PRELIMINARIES

Let $(\Omega, f, \wp)$ be a probability space and let T be an arbitrary set known as (index set). The collection of random variables X = {X: t$\in$T} is defined on $(\Omega, f, \wp)$ is a stochastic process that has an index set. For a time, tX(t) is called the state of the procedure at the time t. Since a stochastic process is a relation of random variables, its requirement is similar to that for random vectors. It could be seen as a statistical event that evolves with time concerning probabilistic laws. In a stochastic process, the collection of random variables is ordered in time and defined at a set of time points which will be continuous or discrete.

Thus, let be the price of some risky asset at a time and μ, an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock follows a random walk which is governed by a stochastic differential equation:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \tag{1.1}$$

where, μ is drift and the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp), \xi$ is σ-algebra generated W_t, $t \geq 0$.

Theorem 1.1: (Ito's formula) Let a filtered probability space X= {X, t≥0} be an adaptive stochastic process on (Ω, β, α, F(β)) possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt+f(t, X(t))dW(t) \tag{1.2}$$

$$t \in \Re \text{ and for } u = u(t, X(t)) \in C^{1\times 2} (\prod \times \mathbb{R}))$$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2} f^2 \frac{\partial^2 u}{\partial x^2}\right\} d\tau + f\frac{\partial u}{\partial x} dW(t) \tag{1.3}$$

Using theorem 1.1 and Eq. 1.1 comfortably solves the SDE with a solution given below:

$$S(t) = S_0 \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1]$$

From Eq. 1.1 Black-Scholes Partial Differential Equation (PDE) is given as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{1.4}$$

Following the method:

$$dS(t) = \hat{\alpha}S(t)dt + \sigma S(t)dW(t), \hat{\alpha} = \alpha + \lambda \tag{1.5}$$

Where is the market price of risk? The equation governing stock options is the backward Black-Scholes partial differential equation given (in one variable):

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (\alpha - \lambda)S\frac{\partial V}{\partial S} - ru = 0 \tag{1.6}$$

Assume λ = 0, Eq. 1.6 gives:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \alpha S\frac{\partial V}{\partial S} - rV = -\frac{\partial V}{\partial t} \tag{1.7}$$

From Eq. 1.7, V≠V(t), hence is a function of alone and we arrived at the following 2nd order nonhomogeneous differential equation by also defining the index price of the form: $-\varphi - GR_t$ which replaces RHS of Eq. 1.7:

$$\frac{1}{2}\sigma^2 S^2 \frac{d^2 V}{dS^2} + \alpha S\frac{dV}{dS} - rV = -\varphi - GR_t \tag{1.8}$$

where, φ represents constant and GR_t is the growth of the underlying asset.

Suppose the interest rate on the left-hand side of Eq. 1.8 becomes:

$$\frac{1}{2}\sigma^2 S^2 \frac{d^2 V}{dS^2} + \alpha S\frac{dV}{dS} - V = -\varphi - GR_t \tag{1.9}$$

Dividing the homogeneous part of Eq. 1.9 gives:

$$S\frac{d^2V}{dS^2}+\frac{2\alpha}{\sigma}\frac{dV}{dS}-\frac{2rV}{S\sigma^2}=0 \tag{1.10}$$

Suppose the trading of shares or adjustment of the portfolio is allowed to take place. We, therefore, adjust the stock variables following the method by setting:

$$\frac{2\alpha}{\sigma}=1,\ \ \frac{2r}{S\sigma^2}=S\sigma^2V$$

Since the rate of change of investment output value depends on the investment output at the present time t, (Eq. 1.10) becomes:

$$S\frac{d^2V}{dS^2}+\frac{dV}{dS}-S\sigma^2V=0 \tag{1.11}$$

The last term of LHS of (1.11) is the aggregate intrinsic value of the stock, hence it comprises the volatility of the underlying asset throughout the trading days. Secondly, risk factors S describe any kind of risk and uncertainty present in the financial market such as stock prices, interest rates, strike price, etc. So combining (1.11) and (LHS) of (1.10) gives a complete 2nd non-homogenous ordinary differential equation as follows:

$$S\frac{d^2V}{dS^2}+\frac{dV}{dS}-S\sigma^2V=-\varphi-GR_t \tag{1.12}$$

MATHEMATICAL FORMULATION

Here investors in all seasons receive their wages at different time intervals. Assuming the rate of return grows in two different ways namely: Multiplicative effects series and additive effect series, at a time t with a quadratic function, which is characterized by fluctuations due to some environmental effects. What will be the investor's expectation in terms of stock returns and the value of assets? Hence, we consider two different investments selected stocks (companies) to be represented as S_1 and S_2 defined in the system of coupled variable coefficient differential equations. Therefore, the rate of returns is defined as:

$$R_t\!:=(\lambda_1\lambda_2)^2,\ldots \text{ where, } t=1,2\ldots \tag{1.13}$$

$$R_t\!:=(\lambda_1+\lambda_2)^2,\ldots \text{ where, } t=1,2\ldots \tag{1.14}$$

Equation 1.13 and 1.14 represent stock return with multiplicative effects series and additive effects series, respectively. Using Eq. 1.13 and 1.14 in Eq. 1.12 independently gives the following investment equations for two different stocks (companies):

$$S_1\frac{d^2V_{1(t)}}{dS_1^{\;2}}+\frac{dV_{1(t)}}{dS_1}-(\lambda_1\lambda_2)^2S_1\sigma^2V_{1(t)}=-\varphi-GR_tV_{2(t)} \tag{1.15}$$

$$S_2\frac{d^2V_{2(t)}}{dS_2^{\;2}}+\frac{dV_{2(t)}}{dS_2}-(\lambda_1+\lambda_2)^2S_2\sigma^2V_{2(t)}=0 \tag{1.16}$$

With the following boundary conditions:

$$\left.\begin{aligned} & V_{1(t)} = 0, \text{ on } S = 1 \\ & \frac{dV_{1(t)}}{dS_1} = \frac{dV_{2(t)}}{dS_2} = 0 \end{aligned}\right\} \tag{1.17}$$

Method of solution: The proposed model Eq. 1.15 to 1.17 consists of a system of variable coefficient financial differential equations whose solutions are not trivial. We adopt the method of Frobenius in solving the homogenous part of the problem. To grab this problem adequately we note that $V_{1(t)}(S)$, $V_{2(t)}(S) < \infty$ for all $S \in [0,1]$ from Eq. 1.16:

Let:

$$V_{2(t)} = \sum_{m=0}^{\infty} a_m S_2^{m+c} = S_2^c \sum_{m=0}^{\infty} a_m S_2^m \tag{1.18}$$

$$V'_{2(t)} = \sum_{m=0}^{\infty} a_m (m+c) S_2^{m+c-1} = S_2^{c=1} \sum_{m=0}^{\infty} a_m (m+c) S_2^m \tag{1.19}$$

$$V''_{2(t)} = S_2^{c-2} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_2^m = S_2^{c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_2^m \tag{1.20}$$

Substituting Eq. 1.18-1.20 into Eq. 1.16 and carrying out some algebraic simplifications gives:

$$V_{2(t)}(S_2) = S_2^{\,c} \left\{ \begin{aligned} & a_0 + \frac{(\lambda_1+\lambda_2)^2 \sigma^2 a_0 S_2^{\,2}}{(c+2)^2} + \frac{(\lambda_1+\lambda_2)^4 \sigma^4 a_0 S_2^{\,4}}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1+\lambda_2)^6 \sigma^6 a_0 S_2^{\,6}}{(c+2)^2 (c+4)^2 (c+6)^2} \\ & + \frac{(\lambda_1+\lambda_2)^8 \sigma^8 a_0 S_2^{\,8}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1+\lambda_2)^{10} \sigma^{10} a_0 S_2^{\,10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} + \ldots \end{aligned} \right\}$$

That is:

$$V_{2(t)}(S) = a_0 S^c \left\{ \begin{aligned} & 1 + \frac{(\lambda_1+\lambda_2)^2 \sigma^2 a_0 S_2^{\,2}}{(c+2)^2} + \frac{(\lambda_1+\lambda_2)^2 \sigma^4 a_0 S_2^{\,4}}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1+\lambda_2)^2 \sigma^6 a_0 S_2^{\,6}}{(c+2)^2 (c+4)^2 (c+6)^2} \\ & + \frac{(\lambda_1+\lambda_2)^2 \sigma^8 a_0 S_2^{\,8}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1+\lambda_2)^2 \sigma^{10} a_0 S_2^{\,10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} \end{aligned} \right\} \tag{1.21}$$

When c = 0 (Eq. 1.21) becomes:

$$V_{2(t)}(S_2) = v_1 = \phi_1 \left\{ \begin{aligned} & 1 + \frac{(\lambda_1+\lambda_2)^2 \sigma^2 S_2^{\,2}}{2^2} + \frac{(\lambda_1+\lambda_2)^4 \sigma^4 S_2^{\,4}}{2^2 \times 4^2} + \frac{(\lambda_1+\lambda_2)^6 \sigma^6 S_2^{\,6}}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1+\lambda_2)^8 \sigma^8 S_2^{\,8}}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ & + \frac{(\lambda_1+\lambda_2)^{10} \sigma^{10} S_2^{\,10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{aligned} \right\} \tag{1.22}$$

Another is given by:

$$v_2 = \frac{dV_{2(t)}}{dc}$$

$$\frac{dV_{2(t)}}{dc}=a_0S_2^c\ln r\left\{1+\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{(c+2)^2}+\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{(c+2)^2(c+4)^2}+\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{(c+2)^2(c+4)^2(c+6)^2}\right.$$
$$\left.+\frac{(\lambda_1+\lambda_2)^8\sigma^8S_2^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2(c+10)^2}..\right\}$$
$$+a_0S_2{}^c\frac{d}{dc}\left\{1+\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{(c+2)^2}+\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{(c+2)^2(c+4)^2}+\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{(c+2)^2(c+4)^2(c+6)^2}\right.$$
$$\left.+\frac{(\lambda_1+\lambda_2)^8\sigma^8S_2^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S^{10}}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2(c+10)^2}+...\right\}$$

$$\frac{dV_{2(t)}}{dc}=a_0S_2^c\ln S\left\{1+\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{(c+2)^2}+\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2{}^4}{(c+2)^2(c+4)^2}+\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{(c+2)^2(c+4)^2(c+6)^2}\right.$$
$$\left.+\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2(c+10)^2}+...\right\}$$
$$+a_0r^c\left\{0-\frac{2(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{(c+2)^3}-\frac{4(\lambda_1+\lambda_2)^4\sigma^4S_2^4(c+3)}{(c+2)^3(c+4)^3}-\frac{8(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{(c+2)^3(c+4)^3(c+6)^3}\right.$$
$$\left.+\frac{16(\lambda_1+\lambda_2)^8\sigma^8S^8}{(c+2)^3(c+4)^3(c+6)^3(c+8)^3}-\frac{32(\lambda_1+\lambda_2)^{10}\sigma^{10}S^{10}}{(c+2)^3(c+4)^3(c+6)^3(c+8)^3}+...\right\}$$

When c = 0:

$$V_{2(t)}=v_2=\phi_2\left\{\begin{array}{l}\ln S_2\left(\begin{array}{l}1+\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{2^2}+\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{2^2\times4^2}+\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{2^2\times4^2\times6^2}+\frac{(\lambda_1+\lambda_2)^8\sigma^8S_2^8}{2^2\times4^2\times6^2\times8^2}\\+\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}\end{array}\right)\\+a_0S_2{}^c\left(\begin{array}{l}-\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{2^2}-\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{2^3\times4^2}-\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{4^3\times6^3}+\frac{(\lambda_1+\lambda_2)^8\sigma^8S_2^8}{2^5\times6^3\times8^3}\\-\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{4^2\times6^3\times8^3\times10^3}+...\end{array}\right)\end{array}\right\}\tag{1.23}$$

A linear combination Eq. 1.22 and Eq. 1.23 gives the complete solution:

$$\begin{array}{l}V_{2(t)}(S_2)=\phi_1\left\{\begin{array}{l}1+\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{2^2}+\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{2^2\times4^2}+\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{2^2\times4^2\times6^2}+\frac{(\lambda_1+\lambda_2)^8\sigma^8S_2^8}{2^2\times4^2\times6^2\times8^2}\\+\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}+...\end{array}\right\}\\+\phi_2\left\{\begin{array}{l}\ln S_2\left(\begin{array}{l}1+\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{2^2}+\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{2^2\times4^2}+\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{2^2\times4^2\times6^2}+\frac{(\lambda_1+\lambda_2)^8\sigma^8S_2^8}{2^2\times4^2\times6^2\times8^2}\\+\frac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}\end{array}\right)\\-\frac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{2^2}-\frac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{2^3\times4^2}-\frac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{4^3\times6^3}+\frac{(\lambda_1+\lambda_2)^8\sigma^8S_2^8}{2^5\times6^3\times8^3}\\-\frac{(\lambda_1+\lambda_2)^2\sigma^{10}S_2^{10}}{4^2\times6^3\times8^3\times10^3}+...\end{array}\right\}\end{array}\tag{1.24}$$

Applying the boundary conditions (Eq. 1.17) and setting them gives:

$$\phi_1=\frac{V_{2a}}{y_1(1)}$$

Where:

$$y_1(1)=\left\{\begin{matrix}1+\dfrac{(\lambda_1+\lambda_2)^2\sigma^2}{2^2}+\dfrac{(\lambda_1+\lambda_2)^4\sigma^4}{2^2\times4^2}+\dfrac{(\lambda_1+\lambda_2)^6\sigma^6}{2^2\times4^2\times6^2}+\dfrac{(\lambda_1+\lambda_2)^8\sigma^8}{2^2\times4^2\times6^2\times8^2}\\+\dfrac{(\lambda_1+\lambda_2)^{10}\sigma^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}+\dots\end{matrix}\right\}$$

$$V_{2(t)}(S_2)=\frac{V_{2a}}{y_1(1)}\left\{\begin{matrix}1+\dfrac{(\lambda_1+\lambda_2)^2\sigma^2S_2^2}{2^2}+\dfrac{(\lambda_1+\lambda_2)^4\sigma^4S_2^4}{2^2\times4^2}+\dfrac{(\lambda_1+\lambda_2)^6\sigma^6S_2^6}{2^2\times4^2\times6^2}+\dfrac{(\lambda_1+\lambda_2)^6\sigma^8S_2^8}{2^2\times4^2\times6^2\times8^2}\\+\dfrac{(\lambda_1+\lambda_2)^{10}\sigma^{10}S_2^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}+\dots\end{matrix}\right\} \tag{1.25}$$

From the homogenous part of Eq. 1.15.

Let:

$$V_{1(t)}=\sum_{m=0}^{\infty}a_mS_1^{m+c}=S_1^{c}\sum_{m=0}^{\infty}a_mS_1^{m} \tag{1.26}$$

$$V'_{1(t)}=\sum_{m=0}^{\infty}a_m(m+c)S_1^{m+c-1}=S_1^{c=1}\sum_{n=0}^{\infty}a_m(m+c)S_1^{m} \tag{1.27}$$

$$V''_{1(t)}=S_1^{c-2}\sum_{m=0}^{\infty}a_m(m+c)(m+c-1)S_1^{m}=S_1^{c-1}\sum_{m=0}^{\infty}a_m(m+c)(m+c-1)S_1^{m} \tag{1.28}$$

Substituting Eq. 1.26-1.28 into Eq. 1.15 gives:

$$V_{2(t)}(S_1)=S^c\left\{\begin{matrix}a_0+\dfrac{(\lambda_1\lambda_2)^2\sigma^2a_0S^2}{(c+2)^2}+\dfrac{(\lambda_1\lambda_2)^4\sigma^4a_0S^4}{(c+2)^2(c+4)^2}+\dfrac{(\lambda_1\lambda_2)^6\sigma^6a_0S^6}{(c+2)^2(c+4)^2(c+6)^2}\\+\dfrac{(\lambda_1\lambda_2)^8\sigma^8a_0S^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\dfrac{(\lambda_1\lambda_2)^{10}\sigma^{10}a_0S^{10}}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2(c+10)^2}+\dots\end{matrix}\right\}$$

That is:

$$V_{1(t)}(S)=a_0S^c\left\{\begin{matrix}1+\dfrac{(\lambda_1\lambda_2)^2\sigma^2a_0S^2}{(c+2)^2}+\dfrac{(\lambda_1\lambda_2)^4\sigma^4a_0S^4}{(c+2)^2(c+4)^2}+\dfrac{(\lambda_1\lambda_2)^6\sigma^6a_0S^6}{(c+2)^2(c+4)^2(c+6)^2}\\+\dfrac{(\lambda_1\lambda_2)^8\sigma^8a_0S^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\dfrac{(\lambda_1\lambda_2)^{10}\sigma^{10}a_0S^{10}}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2(c+10)^2}\end{matrix}\right\} \tag{1.29}$$

When c = 0 (Eq. 1.29) becomes:

$$V_{1(t)}(S)=v_1=\alpha_1\left\{\begin{matrix}1+\dfrac{(\lambda_1\lambda_2)^2\sigma^2S^2}{2^2}+\dfrac{(\lambda_1\lambda_2)^4\sigma^4S^4}{2^2\times4^2}+\dfrac{(\lambda_1\lambda_2)^6\sigma^6S^6}{2^2\times4^2\times6^2}+\dfrac{(\lambda_1\lambda_2)^8\sigma^8S^8}{2^2\times4^2\times6^2\times8^2}\\+\dfrac{(\lambda_1\lambda_2)^{10}\sigma^{10}S^{10}}{2^2\times4^2\times6^2\times8^2\times10^2}+\dots\end{matrix}\right\} \tag{1.30}$$

Another is given by:

$$v_2 = \frac{dV_{1(t)}}{dc}$$

$$\begin{aligned}\frac{dV_{1(t)}}{dc} = {} & a_0 S^c \ln r \left\{ 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right. \\ & \left. + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S^8}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} .. \right\} \\ & + a_0 S^c \frac{d}{dc} \left\{ 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right. \\ & \left. + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S^8}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} + \ldots \right\}\end{aligned}$$

$$\begin{aligned}\frac{dV_{1(t)}}{dc} = {} & a_0 S^c \ln S \left\{ 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right. \\ & \left. + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} + \ldots \right\} \\ & + a_0 r^c \left\{ 0 - \frac{2(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^3} - \frac{4(\lambda_1\lambda_2)^4 \sigma^4 S^4 (c+3)}{(c+2)^3 (c+4)^3} - \frac{8(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^3 (c+4)^3 (c+6)^3} + \frac{16(\lambda_1\lambda_2)^8 \sigma^8 S^8}{(c+2)^3 (c+4)^3 (c+6)^3 (c+8)^3} \right. \\ & \left. - \frac{32(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^3 (c+4)^3 (c+6)^3 (c+8)^3} + \ldots \right\}\end{aligned}$$

When c= 0:

$$V_{1(t)} = v_2 = \alpha_2 \left\{ \begin{aligned} & \ln S_1 \left(\begin{aligned} & 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^2}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^4}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^6}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ & + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{aligned} \right) \\ & + a_0 S_1^c \left(\begin{aligned} & - \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^2}{2^2} - \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^4}{2^3 \times 4^2} - \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^6}{4^3 \times 6^3} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^8}{2^5 \times 6^3 \times 8^3} \\ & - \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \ldots \end{aligned} \right) \end{aligned} \right\} \tag{1.31}$$

A linear combination of Eq. 1.30 and Eq. 1.31 gives the complete solution:

$$\begin{aligned} V_{1(t)}(S_1) = {} & \alpha_1 \left\{ \begin{aligned} & 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^2}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^4}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^6}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ & + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{aligned} \right\} \\ & + \alpha_2 \left\{ \begin{aligned} & \ln S_1 \left(\begin{aligned} & 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^2}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^4}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^6}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ & + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{aligned} \right) \\ & - \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^2}{2^2} - \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^4}{2^3 \times 4^2} - \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1{}^6}{4^3 \times 6^3} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^8}{2^5 \times 6^3 \times 8^3} - \frac{(\lambda_1\lambda_2)^2 \sigma^{10} S_1^{10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \ldots \end{aligned} \right\} \end{aligned} \tag{1.32}$$

Applying the boundary conditions in Eq. 1.17 and setting $\alpha_2 = 0$ gives:

$$V_{1(t)}(S_1) = \left\{ \begin{aligned} & 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^2}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^4}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^6}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^8 S_1^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ & + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{aligned} \right\} \tag{1.33}$$

To get the particular solution of Eq. 1.15 which is the non-homogenous part of the problem, we solve using the method of undetermined coefficients:

$$y(S_2)=\phi_1\left\{\begin{array}{l}1+\dfrac{(\lambda_1+\lambda_2)^2S_2^2}{2^2}+\dfrac{(\lambda_1+\lambda_2)^4S_2^4}{2^2\times 4^2}+\dfrac{(\lambda_1+\lambda_2)^6S_2^6}{2^2\times 4^2\times 6^2}+\dfrac{(\lambda_1+\lambda_2)^8S_2^8}{2^2\times 4^2\times 6^2\times 8^2}\\+\dfrac{(\lambda_1+\lambda_2)^8S_2^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots\end{array}\right\},A\in R$$

The complementary function for Eq. 1.16 is:

$$V_{1(t)c}(S)=C_c\left[\begin{array}{l}1+\dfrac{(\lambda_1\lambda_2\sigma S_1)^2}{2^2}+\dfrac{(\lambda_1\lambda_2\sigma S_1)^4}{(2\times 4)^2}+\dfrac{(\lambda_1\lambda_2\sigma S_1)^6}{(2\times 4\times 6)^2}+\dfrac{(\lambda_1\lambda_2\sigma S_1)^8}{(2\times 4\times 6\times 8)^2}\\+\dfrac{(\lambda_1\lambda_2\sigma S_1)^{10}}{(2\times 4\times 6\times 8\times 10)^2}+\ldots\end{array}\right] \tag{1.34}$$

Consider the particular solution:

$$V_p(S_1)=A_0+A_1S_1^2+A_2S_1^4+A_3S_1^6+A_4S_1^8+A_5S_1^{10} \qquad (1.35)\ (3.5.12)$$

$$V_p'(S_1)=2A_1S_1+4A_2S_1^3+6A_3S_1^5+8A_4S_1^7+10A_5S_1^9 \qquad (1.36)\ (3.5.13)$$

$$V_p''(S_1)=2A_1+12A_2S_1^2+30A_3S_1^4+56A_4S_1^6+90A_5S_1^8 \qquad (1.37)\ (3.5.14)$$

Putting Eq. 1.35-1.37 into Eq. 1.15 after performing some long algebraic expressions gives:

$$V(S_1)=\left[c+\frac{c(\lambda_1\lambda_2\sigma)^2S_1^2}{4}+\frac{c(\lambda_1\lambda_2\sigma)^4S_1^4}{(2\times 4)^2}+\frac{c(\lambda_1\lambda_2\sigma)^6S_1^6}{(2\times 4\times 6)^2}+\frac{c(\lambda_1\lambda_2\sigma)^8S_1^8}{(2\times 4\times 6\times 8)^2}+\frac{c(\lambda_1\lambda_2\sigma)^{10}S_1^{10}}{(2\times 4\times 6\times 8\times 10)^2}\right]$$
$$+A_0+A_1S_1^2+A_2S_1^4+A_3S_1^6+A_4S_1^8+A_5S_1^{10}$$

Taking like terms gives a complete solution:

$$\begin{aligned}V_{1(t)}(S_1)={}&c+A_0+\left(\frac{c(\lambda_1\lambda_2\sigma)^2}{4}+A_1\right)S_1^2+\left(\frac{c(\lambda_1\lambda_2\sigma)^4}{(2\times 4)^2}+A_2\right)S_1^4+\left(\frac{c(\lambda_1\lambda_2\sigma)^6}{(2\times 4\times 6)^2}+A_3\right)S_1^6\\&+\left(\frac{c(\lambda_1\lambda_2\sigma)^8}{(2\times 4\times 6\times 8)^2}+A_4\right)S_1^8+\left(\frac{c(\lambda_1\lambda_2\sigma)^{10}}{(2\times 4\times 6\times 8\times 10)^2}+A_5\right)S_1^{10}\end{aligned} \tag{1.41}$$

Where are constants seen in Appendix 1?

Appendix 1:

$$\Rightarrow A_5=-\frac{1}{(\lambda_1\lambda_2\sigma)^2}\left[\frac{((\lambda_1+\lambda_2)\sigma)^{10}GR_tV_{2(t)}}{(2\times 4\times 6\times 8\times 10)^2}\right],\quad \Rightarrow A_4=-\frac{1}{(\lambda_1\lambda_2\sigma)^2}\left[100A_5+\frac{((\lambda_1+\lambda_2)\sigma)^8GR_tV_{2(t)}}{(2\times 4\times 6\times 8)^2}\right]$$

$$\Rightarrow A_3=-\frac{1}{(\lambda_1\lambda_2\sigma)^2}\left[64A_4-\frac{(\lambda_1+\lambda_2\sigma)^6GR_tV_{2(t)}}{(2\times 4\times 6)^2}\right],\quad \Rightarrow A_2=-\frac{1}{(\lambda_1\lambda_2\sigma)^2}\left[36A_3-\frac{((\lambda_1+\lambda_2)\sigma)^4GR_tV_{2(t)}}{(2\times 4)^2}\right]$$

$$\Rightarrow A_1=-\frac{1}{(\lambda_1\lambda_2\sigma)^2}\left[16A_2-\frac{((\lambda_1+\lambda_2)\sigma)^2GR_tV_{2(t)}}{4}\right],\quad \Rightarrow A_0=-\frac{1}{(\lambda_1\lambda_2)^2}\left[4A_1+\varphi-GR_tV_{2(t)}\right]$$

RESULTS AND DISCUSSION

In this section, we present the computational results for the problem formulated in Eq. 1.15-1.17 whose solution is in Eq. 1.25 and 1.41. The graphical results are implemented in a mathematical programming language and MATLAB. However, the following parameter values were explicitly used in the simulation study.

The levels of stock returns described when they are fixed using: 2.5, 3.5, 4.5 and 5.5 show the variation of Stock (1) rate of return with volatility parameter. It can be seen that Stock (1) return reduces with increasing stock volatility. This is correct, because when there is high volatility in the market price, investors find it difficult to invest, thereby causing a low rate of return on investment. There will be no investor who will like to maximize loss in investment. Also, it will result in a kind of panic buying and tensions everywhere in stock trading. Visual inspection of the plot shows high levels of price changes in capital investment. This is in line with the results as shown in Fig. 1.[15].

Here the levels of stock returns described when they are fixed using: 0.23, 0.25, 0.27 and 0.29, show the variation of Stock (1) rate of return with growth rate parameter. This is physically consistent because increasing growth rates in an investment automatically grow the financial strength of the investment over time[16,17]. Also, a careful examination of trends shows levels of changes in the value of assets. With this formation, profit-making is inevitable throughout the trading days. This aligns with the works in literature[18]. However, varying the growth rate parameter does not have an automatic influence on the model; that is why the model retains its shape in the assessments of stock return in time-varying investments. That is to say that, profit-making within the period of trading is indexed in millions of naira, hence it grows unboundedly as shown in Fig. 2.

Again the levels of stock returns described when they are fixed using: 45, 50, 55 and 60, show the variation of Stock (2) rate of return with volatility parameter. In this circumstance, there is exponential growth in the investment which means the trading business is indexed with millions of naira over time. This situation can interest investors to be more courageous in their capital investment hence, it grows exponentially. In addition, the variations of volatility parameters did not affect the investment which is why the plot did not change its shape, which implies that under normal circumstances the investment return is certain as shown in Fig. 3.

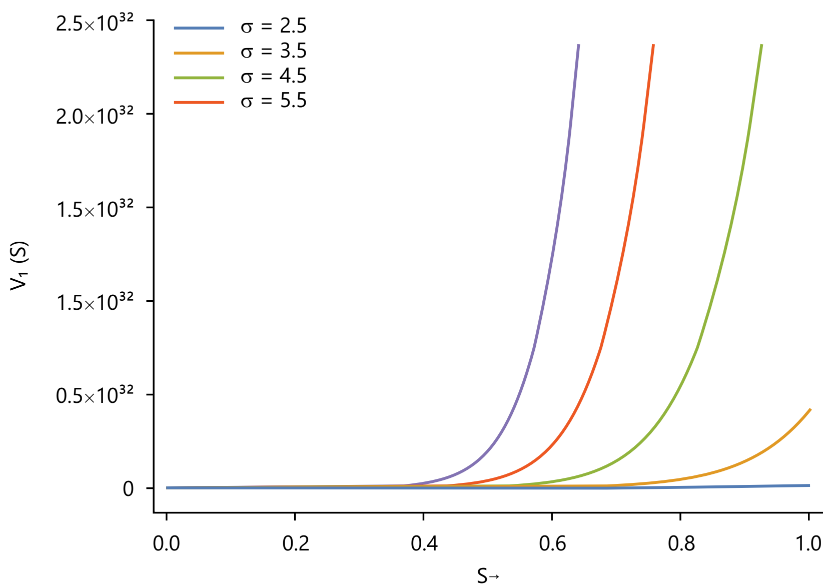

Fig. 1: Stock (1) (S_1) stock return profiles (as seen in the colors) against different values of volatility
Colorful lines compare the profiles of the stock return with different volatility rates

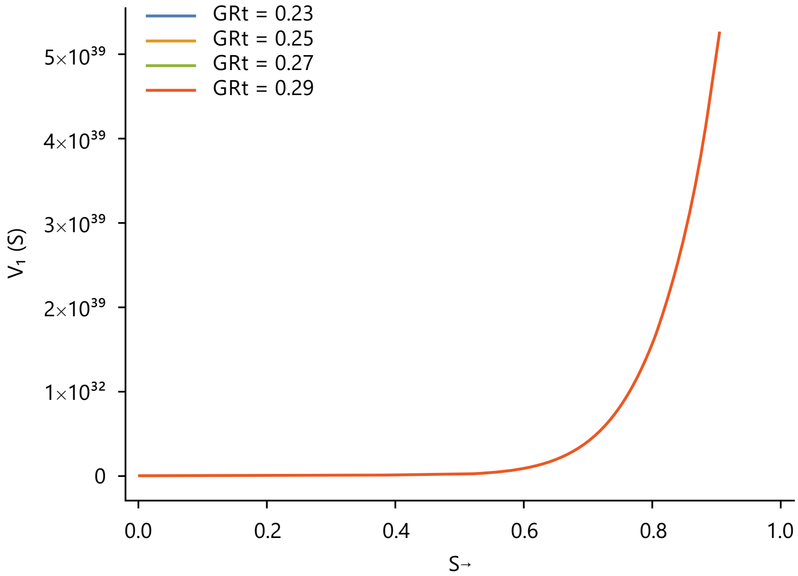

Fig. 2: Stock (1) S_1 trend stock return profiles (as colored) against S different values of Growth rate GR_t
Colorful lines (too tiny to be seen) compare the profiles of the stock return with different growth rates

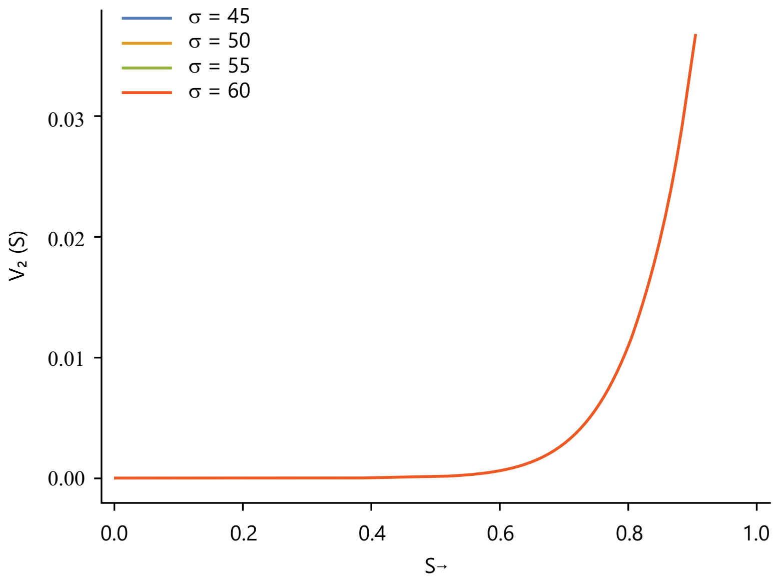

Fig. 3: Stock (2) (S_2) stock return profiles against different values of volatility σ
Colorful lines (shadowed by the orange color) compare the profiles of the return with different volatility rates

$\lambda_1\lambda_2$: This is the multiplicative effect parameter of the model and is used here in company 1 (S_1) to represent stock return rates and value of assets which follow multiplicatively in terms of trading, see columns 2,4 and 6 of Table 1. So when the stock return rates are made constant using: 1.0000, 2.0000 and 3.0000 it responds positively to the value of asset changes which are seen in columns 3,5 and 7. The mean of asset valuation gives the following: 29.4685, 46.6232 and 93.8825. **$\lambda_1+\lambda_2$:** This is the additive effect parameter of the model which is used here in company 2(S_2) to show when stock return rates and in value of assets follow additively, see columns 2,4 and 6 of Table 2. In the same table, when the stock return rates are fixed by 1.0000, 2.0000 and 3.0000 it reflects the changes in the value of assets which are seen in columns 3,5 and 7 while the mean of asset valuation of the second company of our investment equation gives: 29.3594, 46.5470 and 95.6265 for the period of 10 months. These results were in line with the results

Table 1: Multiplicative stock return for assessing the value of assets for the company (1) with the following parameter values: c = 25, σ = 1.3, A_0 = 0.031, A_1 = 0.035, A_2 = 0.016, A_3 = 0.017, A_4 = 0.25 and A_5 = 0.10

S_1	$(\lambda_1\ \lambda_2)$	$V_1\ (S_1)$	$(\lambda_1\ \lambda_2)$	$V_1\ (S_1)$	$(\lambda_1\ \lambda_2)$	$V_1\ (S_1)$
0.1	1.0000	25.1371	2.0000	25.4556	3.0000	25.9911
0.2	1.0000	25.4567	2.0000	26.7512	3.0000	28.9820
0.3	1.0000	25.9940	2.0000	28.9839	3.0000	34.3504
0.4	1.0000	26.7560	2.0000	32.2682	3.0000	42.7232
0.5	1.0000	27.7533	2.0000	36.7740	3.0000	55.0891
0.6	1.0000	29.0008	2.0000	42.7372	3.0000	72.9277
0.7	1.0000	30.5210	2.0000	50.4751	3.0000	98.4034
0.8	1.0000	32.3480	2.0000	60.4094	3.0000	134.6516
0.9	1.0000	34.5374	2.0000	73.0978	3.0000	186.1911
1.0	1.0000	37.1810	2.0000	89.2796	3.0000	259.5152

Table 2: Additive stock return for assessing the value of assets for the company (2) with the following parameter values

S_2	$(\lambda_1+\lambda_2)$	$V_2\ (S_2)$	$(\lambda_1+\lambda_2)$	$V_2\ (S_2)$	$(\lambda_1+\lambda_2)$	$V_2\ (S_2)$
0.1	1.0000	25.1057	2.0000	25.4250	3.0000	25.9600
0.2	1.0000	25.4243	2.0000	26.7175	3.0000	28.9500
0.3	1.0000	25.9590	2.0000	28.9500	3.0000	34.3150
0.4	1.0000	26.7175	2.0000	32.2325	3.0000	42.6875
0.5	1.0000	27.7100	2.0000	36.7325	3.0000	55.0600
0.6	1.0000	28.9500	2.0000	42.6875	3.0000	72.9525
0.7	1.0000	30.4475	2.0000	50.4100	3.0000	98.6825
0.8	1.0000	32.2325	2.0000	60.3150	3.0000	135.8700
0.9	1.0000	34.3150	2.0000	72.9525	3.0000	190.2975
1.0	1.0000	36.7325	2.0000	89.0475	3.0000	271.4900

of Amadi *et al.*[19]. In comparing the mean valuation of the two companies, company 2 has the highest value of 95.6265 but in terms of precision the value of company 1: 93.8825 is better used, which is found in the investment equation of multiplicative effects. This result agrees with literature[20].

More so, in Table 1 and 2, it can be seen that an increase in stock market price dominantly increases the value of an asset. This is quite realistic, because as the cost of goods and services are increasing on a daily basis, will also affect the assets positively. This increase in the value of assets is of good profit margin to investors. This situation can interest investors to be encouraged to invest in a stock exchange business that is indexed in millions of naira. The price changes can be attributed to a lot of influences on the economy over time.

In the coupled systems, the second investment equation with additive effect series has a positive influence over the first investment equation during the trading periods in time-varying investment returns.

CONCLUSION

This paper studied coupled systems of variable coefficient problems with stochastic parameters in the model for two different investments in capital markets. The analytical solutions were solved and the results were obtained as follows: Stock return reduces with increasing stock volatility, an increase in stock market price dominantly increases the value of the assets as stock returns continue to increase, the values of the assets grow exponentially and second investment equation with additive effect series has a positive influence over the first investment equation during the trading periods. By adopting the Frobenius method of series solution and the method of the undetermined coefficient to determine the assessments of stock returns and value of asset prices in time-varying investment returns, a coupled system of variable coefficient problems with stochastic parameters in the model for two different investments in the capital market was studied. The equations were solved as a coupled investment equation when it follows multiplicative, additive effects series with quadratic functions which is not found in the literature.

SIGNIFICANCE STATEMENT

By incorporating multiplicative effect series and additive effects series and solving the problem as coupled systems to establish the behavior of stock return rates and asset price valuation, the proposed model is significant to address the issues in the financial market for an assessment of asset pricing and other capital investments. It will also realistically assess asset value which follows multiplicative and additive effects, respectively, to inform investors to make a viable decision based on the levels of their investment. This study will assist investors or government (as well) to adequately understand the best rate of return using multiplicative and additive effects to influence pricing effects that follow a particular stock return so as so to enhance the effectiveness of their trading business.

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